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The Use Of Progressive Operating Characteristic Values For Gamma And Weibull Distributions On Designing Acceptance Sampling Plan

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Abstract

Operating Characteristic values describes how well an acceptance plan discriminates between good and bad lots. Acceptance sampling plan consists of a sample size n, and the maximum number of defective items that can be found in the sample. Therefore, this paper lead to the establishment of the minimum sample size necessary to assure the life of a product under Weibull and Gamma distributions and further, the operating characteristic values are computed from the sample obtained. Hence, in comparing the performance of the two underlined distributions, it can be easily observed that Weibull distribution performs better than the Gamma distribution in minimizing the failures out of the sample units and the producer's risk.

Keywords:- Acceptance sampling plan, Operating characteristics, Weibull distribution, Gamma distribution.

Introduction

Acceptance sampling is an inspecting procedure applied in statistical quality control. It is a method of measuring random samples of populations called "lots" of materials or products against predetermined standards (Juran and Gryna, 1988). Acceptance

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sampling is a part of operations management or accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management. Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. The products may be grouped into batches or lots or may be single pieces from a continuous operation. A random sample is selected and could be checked for various characteristics (Montgomery, 2005).

Acceptance sampling is part of the process between a part's producer and consumer.

To help determine the quality of a process (or lot) the producer or consumer can take a sample instead of inspecting the full lot. Sampling reduces costs, because one needs to inspect or testfewer items than looking at the whole lot (Goode and Kao, 2012).

The operating characteristic (OC) is the primary tool in lot acceptance sampling plans (LASP). They allow data from a sample to be used to draw conclusions about the lot as a whole with defined risks. One limitation of this methodology is that it only applies to a task that is performed repetitively at the same conditions because it requires all of the samples to be taken from the same population. OC curves allow the test planners to easily see how changing different criteria that define required performance and level of risk or sample size impacts the required amount of testing. Therefore, they are an ideal way to balance cost and risk. Understanding OC curves can also be a practical way to better understanding of alpha, beta, and sample size for many types of lifetime distributions (Aslam and Jun, 2009).

As stated in Muhammad *et al.* (2014) one should be cautious in using group acceptance sampling plan when the sample size is small because design parameters are derived from the asymptotic distribution.

Sample size and the length of experiment time are two major factors regarding cost in a life test. How to design an acceptance sampling scheme to conduct a life test in a short time with small sample size is the main concern for experimenters. Typically, ordinary acceptance sampling plans regarding lifetimes are developed to satisfy two points on the operating characteristic curve (OCC) to protect producers and consumers simultaneously or based on the economic viewpoint (Muhammad et al., 2014).

The acceptance sampling can be used to save time and cost and for more strict inspection of submitted product. In this paper, we develop sampling plan based on the operating characteristic values assuming that the lifetimes of a product follows the Weibull and Gamma distributions with known shape parameters. The chance of rejecting a good item is the Producer's risk whereas the chance of accepting a bad item is called the Consumer's risk denoting by α and β respectively. A risk is involved because an entire lot of material is not being inspected, not everything is known, so sampling will always incur certain risks (Wadsworth *et al.,* 2002). This incurs the risk of making two types of errors as:

A lot may be rejected that should be accepted and the risk of doing this is the producer's risk.

The second error is that a lot may be accepted that should have been rejected and the risk of doing this is called the consumer's risk.

Materials and Methods

The data utilized for this research work were simulated using R www.cran.r.project.org statistical package. The values for the parameters of the distributions are assumed. In this investigation, we consider Weibull and Gamma distributions to assess the producer's risk and failures occurred from the operating characteristic values when changing the sample size. Because, they can be used to model life times of technical systems with repeated repairing after failure (Wadsworth *et al.,* 2002). Further, they adequately describes observed failures of many different types of components and phenomenon. Besides, they incorporate the properties of some lifetime distributions such as exponential distribution.

Binomial Distribution

In order to obtain the minimum sample size, we consider the binomial distribution. The problem is to determine the smallest value of n that will satisfy the given values of $(0 < p < 1)$, with sample size (n), acceptance number (c), and $1-\alpha$ is the probability of acceptance.

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1-\alpha \qquad (2.1)$$

Where p is the probability of a failure during the time t given a specified lifetime. If = p is relatively small and n is large the binomial probability may be approximated by Poission Probability with parameter λ=np

Weibull Distribution

The Weibull distribution has been originally defined by the Swedish physicist Waloddi Weibull. He made use of it in Weibull (1939) in connection with the breaking strength of materials. The Weibull distribution is one of the best-known lifetime distributions. It adequately describes observed failures of many different types of components and phenomenon. The Weibull probability distribution of survival times is defined by two parameters; a parameter (denoted k) called the scale parameter and a parameter (denoted λ) called the shape parameter (Weibull, 1939).

The pdf, f(x) of a two-parameter Weibull distribution is given by

$$f(t;\lambda,k) = \begin{cases} \frac{k}{\lambda^k} t^{k-1} e^{-\left(\frac{t}{\lambda}\right)^k}, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad (2.2);$$

where the shape parameter $(k > 0)$ and λ is the scale parameter $(\lambda > 0)$

Gamma distribution

A random variable T is gamma distributed with parameter k and λ $(T \sim \Gamma(k,\lambda))$, if the following holds: The probability density function is given as

$$f(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{\Gamma(k)} \quad k,\lambda > 0 \qquad (2.3)$$

where λ is the scale parameter and k is the shape parameter

If k = 1, the gamma distribution is reduced to the exponential distribution.

With integer k, the gamma distribution is sometimes called a special Erlangian distribution. It can be derived as the distribution of waiting time to the k-th emission from a Poisson source with intensity parameter λ. Consequently, the sum of k independent exponential variates with parameter λ has a gamma distribution with parameters k and λ and can be used to model life times of technical systems with repeated repairing after failure (Zografos and Balakrishnan, 2009).

Operating Characteristic of the Sampling Plan

The operating characteristic (OC) function of the sampling plan (n, c) is the probability of accepting a lot. It is given by:

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{2.4}$$

where c is the acceptance number, t/λ is the true mean life and p is the probability of failure that will satisfy the following inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha \tag{2.5}$$

where 1-α is the probability of acceptance.

Results and Discussion

The result obtained in this research is presented in Tables 1, 2, 3, and 4 subsequently, so as to compare the operating characteristics of the sampling plan under Weibull and Gamma distributions. Table 1, however, presented the minimum sample sizes necessary to assert the average life to exceed a given true mean life (t/λ_o) with probability $P^{\times}$ and the corresponding acceptance number C using Binomial probabilities.

Table 1: Minimum sample sizes with probability p and the corresponding acceptance number c using Binomial probabilities

P		True mean life (t/λ_o)				
0.75	C	1.0	1.25	1.50	1.75	2.00
	0	4	2	2	2	1
	1	7	5	4	3	3
	2	10	7	5	5	4
	3	13	9	7	6	6
	4	16	11	9	8	7
	5	19	13	11	9	9
0.90	0	6	4	3	2	2
	1	10	6	5	4	4
	2	13	9	7	6	5
	3	17	11	9	7	7
	4	20	13	11	9	8
	5	23	16	12	11	10
0.95	0	7	4	3	3	2
	1	11	7	6	5	4
	2	15	10	8	7	6
	3	19	13	10	8	7
	4	23	15	12	10	9
	5	26	17	14	12	10
0.99	0	11	7	5	4	4
	1	16	10	8	6	5
	2	20	13	10	8	7
	3	24	16	12	10	9
	4	28	18	14	12	11
	5	32	21	16	14	12

We can get the test termination time true mean life (k/λ) for various choices of c, and t/λ_o in order that the producer's risk may not exceed 0.05. Thus, from Table 1, considering the minimum value of the parameters involved (c, $p^{\times}$ and t/λ_o) i.e when the true mean life t/λ_o = 1.0, with the probability of failure $p^{\times} = 0.75$ at acceptance number c = 0, we obtained the minimum sample size n = 4.

Table 2: Operating characteristic values of the sampling plan (n, c, t/λ) for given $P^{\times}$ under Weibull distribution with test termination time (k/λ)

				Test termination time (k/λ)					
$P^{\times}$	C	n	t/λ	2	4	6	8	10	12
0.75	2	10	1.00	0.999	1.000	1.000	1.000	1.000	1.000
		7	1.25	0.987	1.000	1.000	1.000	1.000	1.000
		5	1.50	0.963	1.000	1.000	1.000	1.000	1.000
		5	1.75	0.873	1.000	1.000	1.000	1.000	1.000
		4	2.00	0.856	1.000	1.000	1.000	1.000	1.000
0.90	2	13	1.00	0.998	1.000	1.000	1.000	1.000	1.000
		9	1.25	0.973	1.000	1.000	1.000	1.000	1.000
		7	1.50	0.901	1.000	1.000	1.000	1.000	1.000
		6	1.75	0.796	1.000	1.000	1.000	1.000	1.000
		5	2.00	0.736	1.000	1.000	1.000	1.000	1.000
0.95	2	15	1.00	0.998	1.000	1.000	1.000	1.000	1.000
		10	1.25	0.963	1.000	1.000	1.000	1.000	1.000
		8	1.50	0.861	1.000	1.000	1.000	1.000	1.000
		7	1.75	0.712	1.000	1.000	1.000	1.000	1.000
		6	2.00	0.611	1.000	1.000	1.000	1.000	1.000
0.99	2	20	1.00	0.994	1.000	1.000	1.000	1.000	1.000
		13	1.25	0.926	1.000	1.000	1.000	1.000	1.000
		10	1.50	0.769	1.000	1.000	1.000	1.000	1.000
		8	1.75	0.626	1.000	1.000	1.000	1.000	1.000
		7	2.00	0.491	1.000	1.000	1.000	1.000	1.000

Table 3: Operating characteristic values of the sampling plan (n, c, t/λ°κ) for given $P^{\times}$ under Gamma distribution with test termination time (k/λ)

				Test termination time (k/ λ)					
$P^{\times}$	C	n	t/λ	2	4	6	8	10	12
0.75	2	10	1.00	0.8894	0.9971	0.9997	0.9999	1.0000	1.0000
		7	1.25	0.7683	0.9922	0.9922	0.9999	1.0000	1.0000
		5	1.50	0.8606	0.9956	0.9996	0.9999	1.0000	1.0000
		5	1.75	0.6796	0.9853	0.9984	0.9997	0.9999	1.0000
		4	2.00	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
0.90	2	13	1.00	0.8104	0.9922	0.9995	0.9999	1.0000	1.0000
		9	1.25	0.7683	0.9838	0.9992	0.9997	1.0000	1.0000
		7	1.50	0.6258	0.9853	0.9983	0.9997	0.9999	1.0000
		6	1.75	0.6796	0.9627	0.9984	0.9997	0.9999	1.0000
		5	2.00	0.4657	0.8942	0.9956	0.9961	0.9998	0.9999
0.95	2	15	1.00	0.8104	0.9945	0.9995	0.9999	1.0000	1.0000
		10	1.25	0.7683	0.9922	0.9992	0.9999	1.0000	1.0000
		8	1.50	0.6258	0.9838	0.9983	0.9997	0.9999	1.0000
		7	1.75	0.6796	0.9853	0.9984	0.9997	0.9999	1.0000
		6	2.00	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
0.99	2	20	1.00	0.7280	0.9907	0.9991	0.9998	1.0000	1.0000
		13	1.25	0.5795	0.9816	0.9981	0.9996	0.9999	1.0000
		10	1.50	0.6258	0.9838	0.9983	0.9997	0.9999	1.0000
		8	1.75	0.3675	0.9488	0.9940	0.9988	0.9997	0.9999
		7	2.00	0.3237	0.9627	0.9956	0.9991	0.9998	0.9999

Assume that the life distribution is a Weibull or gamma distribution and that the experimenter is interested in showing that the true unknown average life is at least 1000 hours. From Table 2 and 3, let the consumer's risk be set to $1-P^{x}=0.25$. It is desired to stop the experiment at time t=1000 hours. Then for an acceptance number C=2, the required n is the entry in Tables 2 and 3 corresponding to the values of $1\text{-}P^{x}=0.25$, $t/\lambda_o=1.00$ and C=2. This number is n=10. Thus, n=10 units have to be put on test. If during 1000 hours, no more than 2 failures out of the 10 units are observed, then the experimenter can assert with a confidence level of $P^{x}=0.75$ that the average life is at least 1000 hours.

Comparative Study

Table 4: Operating Characteristic Values for the Sampling Plan (n=10, c=2, $t/\lambda_o=1.00$)

Weibull distribution						
t/λ_o	2	4	6	8	10	12
L(p)	0.999	1.000	1.000	1.000	1.000	1.000
Gamma distribution						
t/λ_o	2	4	6	8	10	12
L(p)	0.8894	0.9971	0.9997	0.9999	1.0000	1.0000

Table 4 shows that, if the true mean life is 8 times of 1000 hours, the Producer's Risk is 0.000. So, a lot of submitted items shall be accepted with probability 0.999 if the true mean life is 2 times the specified mean life. The Accepting Probability of submitted lot is increased up to 1.0000 if the true mean life of an item in a lot is 8 times the specified mean life under Weibull distribution. However, in the case of Gamma distribution it shows that, if the true mean life is 8 times of 1000 hours, the Producer's Risk is 0.0001. So, a lot of submitted items shall be accepted with probability 0.8894 if the true mean life is 2 times the specified mean life. The Accepting Probability of submitted lot is increased up to 0.9999 if the true mean life of an item in a lot is 8 times the specified mean life. Thus, comparing the producer's risk and failures out of the 10 sample units produced by the two underlined distributions, it can be easily observed from table 4 that Weibull distribution perform better than the Gamma distribution in minimizing the producer's risk and failures occurred.

Conclusion

The minimum sample sizes necessary to assure the life of a product under Weibull and Gamma distributions are established in this paper and the operating characteristic values are computed from the samples generated. Thus, in comparing the performance of the two underlined distributions, it can be easily observed that Weibull distribution performs better than the Gamma distribution in minimizing the failures out of the sample units and the producer's risk. It is therefore concluded that the Weibull distribution is more economical and beneficial than the Gamma distribution in terms of minimum sample size, producer's risk and failures occurred.

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